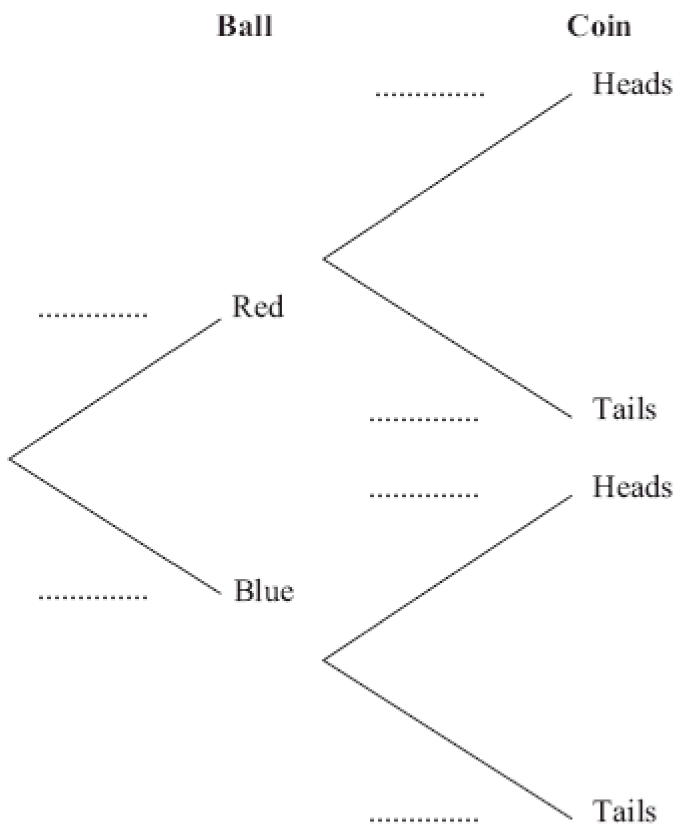
1. An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun.

When a blue ball is selected a fair coin is spun.

(a) Complete the tree diagram below to show the possible outcomes and associated probabilities.



(2)

Shivani selects a ball and spins the appropriate coin.

(b) Find the probability that she obtains a head.

(2)

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

(c) find the probability that Tom selected a red ball.

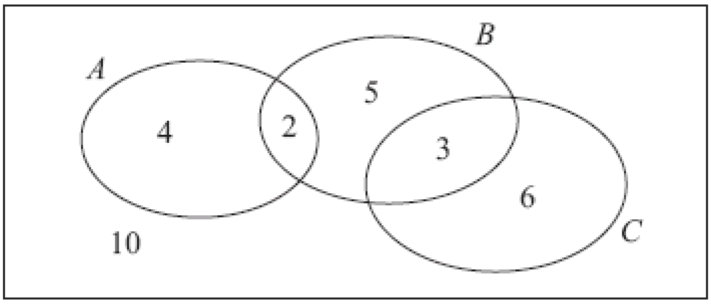
(3)

Shivani and Tom each repeat this experiment.

(d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.

(3)
(Total 10 marks)

2. The Venn diagram below shows the number of students in a class who read any of 3 popular magazines A, B and C.



One of these students is selected at random.

(a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$.

(2)

(b) Find the probability that the student reads A or B (or both).

(2)

(c) Write down the probability that the student reads both A and C.

(1)

Given that the student reads at least one of the magazines,

(d) find the probability that the student reads C.

(2)

(e) Determine whether or not reading magazine B and reading magazine C are statistically independent.

(3)
(Total 10 marks)

3. A jar contains 2 red, 1 blue and 1 green bead. Two beads are drawn at random from the jar without replacement.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. State your probabilities clearly.

(3)

(b) Find the probability that a blue bead and a green bead are drawn from the jar.

(2)
(Total 5 marks)

4. There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

112 take systems support,
70 take developing software,
81 take networking,
35 take developing software and systems support,
28 take networking and developing software,
40 take systems support and networking,
4 take all three extra options.

(a) Draw a Venn diagram to represent this information.

(5)

A student from the course is chosen at random.

Find the probability that this student takes

(b) none of the three extra options,

(1)

(c) networking only.

(1)

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,

(d) find the probability that this student takes all three extra options.

(2)
(Total 9 marks)

5. On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that on a randomly chosen day

(i) Bill travels by foot and is late,

(ii) Bill is not late.

(4)

(c) Given that Bill is late, find the probability that he did not travel on foot.

(4)
(Total 11 marks)

(2)

Shivani selects a ball and spins the appropriate coin.

(b) Find the probability that she obtains a head.

(2)

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

(c) find the probability that Tom selected a red ball.

(3)

Shivani and Tom each repeat this experiment.

(d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.

(3)
(Total 10 marks)

6. (a) Given that P(A) = a and P(B) = b express P($A \cup B$) in terms of a and b when

(i) A and B are mutually exclusive,

(ii) A and B are independent.

(2)

Two events R and Q are such that

P($R \cap Q'$) = 0.15, P(Q) = 0.35 and P($R|Q$) = 0.1

Find the value of

(b) P($R \cup Q$),

(1)

(c) P($R \cap Q$),

(2)

(d) P(R).

(2)
(Total 7 marks)

7. A group of office workers were questioned for a health magazine and $\frac{2}{5}$ were found to take regular exercise. When questioned about their eating habits $\frac{\cancel{3}\,2}{\cancel{2}\,3}$ said they always eat breakfast and, of those who always eat breakfast $\frac{9}{25}$ also took regular exercise. Find the probability that a randomly selected member of the group

(a) always eats breakfast and takes regular exercise,

(2)

(b) does not always eat breakfast and does not take regular exercise.

(4)

(c) Determine, giving your reason, whether or not always eating breakfast and taking regular exercise are statistically independent.

(2)
(Total 8 marks)

8. When Rohit plays a game, the number of points he receives is given by the discrete random variable X with the following probability distribution.

x	0	1	2	3
P($X = x$)	0.4	0.3	0.2	0.1

(a) Find E(X).

(2)

(b) Find F(1.5).

(2)

(c) Show that Var(X) = 1

(4)

(d) Find Var(5 – 3X).

(2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10.
After 3 games he has a total of 6 points.
You may assume that games are independent.

(e) Find the probability that Rohit wins the prize.

(6)
(Total 16 marks)

9. The following shows the results of a wine tasting survey of 100 people.

96 like wine A,
93 like wine B,
96 like wine C,
92 like A and B,
91 like B and C,
93 like A and C,
90 like all three wines.

(a) Draw a Venn Diagram to represent these data.

(6)

Find the probability that a randomly selected person from the survey likes

(b) none of the three wines,

(1)

(c) wine A but not wine B,

(2)

(d) any wine in the survey except wine C,

(2)

(e) exactly two of the three kinds of wine.

(2)

Given that a person from the survey likes wine A,

(f) find the probability that the person likes wine C.

(3)
(Total 16 marks)

10. A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

(a) Find the proportion of students who read both quality and tabloid newspapers.

(3)

(b) Draw a Venn diagram to represent this information.

(3)

A student is selected at random. Given that this student reads newspapers on a regular basis,

(c) find the probability that this student only reads quality newspapers.

(3)
(Total 9 marks)

11. In a factory, machines A, $B$and $C$are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, $B$and C produce 3%, 6% and 5% defective rods respectively.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly selected rod is

(i) produced by machine A and is defective,

(ii) is defective.

(5)

(c) Given that a randomly selected rod is defective, find the probability that it was produced by machine C.

(3)
(Total 11 marks)

12. The random variable X has probability function

$$\mathrm{P}(X = x) = \frac{(2x-1)}{36} \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of X.

(3)

Find

(b) $\mathrm{P}(2 < X \leq 5)$,

(2)

(c) the exact value of $\mathrm{E}(X)$.

(2)

(d) Show that $\mathrm{Var}(X) = 1.97$ to 3 significant figures.

(4)

(e) Find $\mathrm{Var}(2 - 3X)$.

(2)
(Total 13 marks)

13. A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left handed and having dark hair. Glasses were worn by 36 people, 28 were left handed and 36 had dark hair. There were 17 who wore glasses and were left handed, 19 who wore glasses and had dark hair and 15 who were left handed and had dark hair. Only 10 people wore glasses, were left handed and had dark hair.

(a) Represent these data on a Venn diagram.

(6)

A person was selected at random from this group.

Find the probability that this person

(b) wore glasses but was not left handed and did not have dark hair,

(1)

(c) did not wear glasses, was not left handed and did not have dark hair,

(1)

(d) had only two of the attributes,

(2)

(e) wore glasses given that they were left handed and had dark hair.

....

....

....

....

(3)
(Total 13 marks)

14. A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

(a) Draw a tree diagram to represent this information.

(3)

Find the probability that

(b) the second ball selected is red,

(2)

(c) both balls selected are red, given that the second ball selected is red.

(2)
(Total 7 marks)

15. For the events A and B,

$$P(A \cap B') = 0.32, \qquad P(A' \cap B) = 0.11 \qquad \text{and} \qquad P(A \cup B) = 0.65$$

(a) Draw a Venn diagram to illustrate the complete sample space for the events A and B.

(3)

(b) Write down the value of P(A) and the value of P(B).

(3)

(c) Find P($A \mid B'$).

(2)

(d) Determine whether or not A and B are independent.

(3)
(Total 11 marks)

16. In a school there are 148 students in Years 12 and 13 studying Science, Humanities or Arts subjects. Of these students, 89 wear glasses and the others do not. There are 30 Science students of whom 18 wear glasses. The corresponding figures for the Humanities students are 68 and 44 respectively.

A student is chosen at random.

Find the probability that this student

(a) is studying Arts subjects,

(4)

(b) does not wear glasses, given that the student is studying Arts subjects.

(2)

Amongst the Science students, 80% are right-handed. Corresponding percentages for Humanities and Arts students are 75% and 70% respectively.

A student is again chosen at random.

(c) Find the probability that this student is right-handed.

(3)

(d) Given that this student is right-handed, find the probability that the student is studying Science subjects.

(3)
(Total 12 marks)

17. A company assembles drills using components from two sources. Goodbuy supplies 85% of the components and Amart supplies the rest. It is known that 3% of the components supplied by Goodbuy are faulty and 6% of those supplied by Amart are faulty.

(a) Represent this information on a tree diagram.

(3)

An assembled drill is selected at random.

(b) Find the probability that it is not faulty.

(3)
(Total 6 marks)

18. Articles made on a lathe are subject to three kinds of defect, *A*, *B* or *C*. A sample of 1000 articles was inspected and the following results were obtained.

31 had a type *A* defect
37 had a type *B* defect
42 had a type *C* defect
11 had both type *A* and type *B* defects
13 had both type *B* and type *C* defects
10 had both type *A* and type *C* defects
6 had all three types of defect.

(a) Draw a Venn diagram to represent these data.

(6)

Find the probability that a randomly selected article from this sample had

(b) no defects,

(1)

(c) no more than one of these defects.

(2)

An article selected at random from this sample had only one defect.

(d) Find the probability that it was a type *B* defect.

(2)

Two different articles were selected at random from this sample.

(e) Find the probability that both had type B defects.

(2)
(Total 13 marks)

19. The events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.

(a) Using the space below, represent these probabilities in a Venn diagram.

(4)

Hence, or otherwise, find

(b) $P(A \cup B)$,

(1)

(c) $P(A \mid B')$

(2)
(Total 7 marks)

20. A fair die has six faces numbered 1, 2, 2, 3, 3 and 3. The die is rolled twice and the number showing on the uppermost face is recorded each time.

Find the probability that the sum of the two numbers recorded is at least 5.

(Total 5 marks)

21. Three events A, B and C are defined in the sample space S. The events A and B are mutually exclusive and A and C are independent.

(a) Draw a Venn diagram to illustrate the relationships between the 3 events and the sample space.

(3)

Given that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup C) = 0.7$, find

(b) $P(A \mid C)$,

(2)

(c) $P(A \cup B)$,

(2)

(d) $P(C)$.

(4)

(Total 11 marks)

22. The events A and B are such that $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{2}$ and $P(A \mid B') = \frac{4}{5}$.

(a) Find

(i) $P(A \cap B')$,

(ii) $P(A \cap B)$,

(iii) $P(A \cup B)$,

(iv) $P(A \mid B)$.

(7)

(b) State, with a reason, whether or not A and B are

(i) mutually exclusive,

(2)

(ii) independent.

(2)
(Total 11 marks)

23. One of the objectives of a computer game is to collect keys. There are three stages to the game. The probability of collecting a key at the first stage is $\frac{2}{3}$, at the second stage is $\frac{1}{2}$, and at the third stage is $\frac{1}{4}$.

(a) Draw a tree diagram to represent the 3 stages of the game.

(4)

(b) Find the probability of collecting all 3 keys.

(2)

(c) Find the probability of collecting exactly one key in a game.

(5)

(d) Calculate the probability that keys are not collected on at least 2 successive stages in a game.

(5)
(Total 16 marks)

24. A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.

(a) Find the probability that Linda scores 30 points in a round.

(2)

The random variable X is the number of points Linda scores in a round.

(b) Find the probability distribution of X.

(5)

(c) Find the mean and the standard deviation of X.

(5)

A game consists of 2 rounds.

(d) Find the probability that Linda scores more points in round 2 than in round 1.

(6)
(Total 18 marks)

25. Explain what you understand by

(a) a sample space,

(1)

(b) an event.

(1)

Two events A and B are independent, such that P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{4}$.

Find

(c) $P(A \cap B)$,

(1)

(d) $P(A \mid B)$,

(2)

(e) $P(A \cup B)$.

(2)
(Total 7 marks)

26. A car dealer offers purchasers a three year warranty on a new car. He sells two models, the Zippy and the Nifty. For the first 50 cars sold of each model the number of claims under the warranty is shown in the table below.

	Claim	No claim
Zippy	35	15
Nifty	40	10

One of these purchasers is chosen at random. Let A be the event that no claim is made by the purchaser under the warranty and B the event that the car purchased is a Nifty.

(a) Find $P(A \cap B)$.
(2)

(b) Find $P(A')$.
(2)

Given that the purchaser chosen does not make a claim under the warranty,

(c) find the probability that the car purchased is a Zippy.
(2)

(d) Show that making a claim is not independent of the make of the car purchased.

Comment on this result.
(3)
(Total 9 marks)

27. A keep-fit enthusiast swims, runs or cycles each day with probabilities 0.2, 0.3 and 0.5 respectively. If he swims he then spends time in the sauna with probability 0.35. The probabilities that he spends time in the sauna after running or cycling are 0.2 and 0.45 respectively.

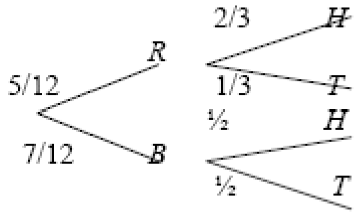
(a) Represent this information on a tree diagram.
(3)

(b) Find the probability that on any particular day he uses the sauna.
(3)

(c) Given that he uses the sauna one day, find the probability that he had been swimming.
(3)

(d) Given that he did not use the sauna one day, find the probability that he had been swimming.
(6)
(Total 15 marks)

1. (a)



P(*R*) and P(*B*)	B1	
2nd set of probabilities	B1	2

Note

1st B1 for the probabilities on the first 2 branches. Accept $0.41\dot{6}$ and $0.58\dot{3}$

2nd B1 for probabilities on the second set of branches. Accept $0.\dot{6}$, $0.\dot{3}$, 0.5 and $\frac{1.5}{3}$

Allow exact decimal equivalents using clear recurring notation if required.

(b) P(*H*) = $\frac{5}{12}\times\frac{2}{3}+\frac{7}{12}\times\frac{1}{2},=\frac{41}{72}$ or awrt 0.569 M1 A1 2

Note

M1 for an expression for P(*H*) that follows through their sum of two products of **probabilities** from their tree diagram

(c) P(R|H) = $\dfrac{\frac{5}{12}\times\frac{2}{3}}{"\frac{41}{72}"}, = \dfrac{20}{41}$ or awrt 0.488 M1 A1ft A1 3

Note

Formula seem

M1 for $\dfrac{\mathrm{P}(R\cap H)}{\mathrm{P}(H)}$ with denominator their (b) substituted e.g.

$\dfrac{\mathrm{P}(R\cap H)}{\mathrm{P}(H)} = \dfrac{\frac{5}{12}}{\text{(their (b))}}$ award M1.

Formula not seen

M1 for $\dfrac{\text{probability}\times\text{probability}}{\text{their } b}$ but M0 if fraction repeated e.g.

$\dfrac{\frac{5}{12}\times\frac{2}{3}}{\frac{2}{3}}$.

1st A1ft for a fully correct expression or correct follow through

2nd A1 for $\frac{20}{41}$ o.e.

(d) $\left(\dfrac{5}{12}\right)^2 + \left(\dfrac{7}{12}\right)^2$ M1 A1ft

$= \dfrac{25}{144} + \dfrac{49}{144} = \dfrac{74}{144}$ or $\dfrac{37}{72}$ or awrt 0.514 A1 3

Note

M1 for $\left(\dfrac{5}{12}\right)^2$ or $\left(\dfrac{7}{12}\right)^2$ can follow through their equivalent values from tree diagram

1st A1 for both values correct or follow through from their original tree and +

2nd A1 for a correct answer

Special Case $\dfrac{5}{12}\times\dfrac{4}{11}$ or $\dfrac{7}{12}\times\dfrac{6}{11}$ seen award M1A0A0s

[10]

2.	(a)	$\frac{2+3}{\text{their total}} = \frac{5}{\text{their total}} = \frac{1}{6}$ **(* * given answer * *)**	M1 A1cso	2
		Note M1 for $\frac{2+3}{\text{their total}}$ or $\frac{5}{30}$		
	(b)	$\frac{4+2+5+3}{\text{total}}$, $= \frac{14}{30}$ or $\frac{7}{15}$ or $0.4\dot{6}$	M1 A1	2
		Note M1 for adding at least 3 of "4, 2, 5, 3" and dividing by their total to give a probability Can be written as separate fractions substituted into the completely correct Addition Rule		
	(c)	$\mathrm{P}(A \cap C) = 0$	B1	1
		Note B1 for 0 or 0/30		
	(d)	$\mathrm{P}(\mathrm{C} \mid \text{reads at least one magazine}) = \frac{6+3}{20} = \frac{9}{20}$	M1 A1	2
		Note M1 for a **denominator of 20** or $\frac{20}{30}$ leading to an answer with denominator of 20 $\frac{9}{20}$ only, 2/2		

(e) $\text{P}(B) = \dfrac{10}{30} = \dfrac{1}{3}$, $\text{P}(C) = \dfrac{9}{30} = \dfrac{3}{10}$, $\text{P}(B \cap C) = \dfrac{3}{30} = \dfrac{1}{10}$

or $\text{P}(B|C) = \dfrac{3}{9}$ M1

$\text{P}(B) \times \text{P}(C) = \dfrac{1}{3} \times \dfrac{3}{10} = \dfrac{1}{10} = \text{P}(B \cap C)$ or $\text{P}(B|C) = \dfrac{3}{9} = \dfrac{1}{3} = \text{P}(B)$ M1

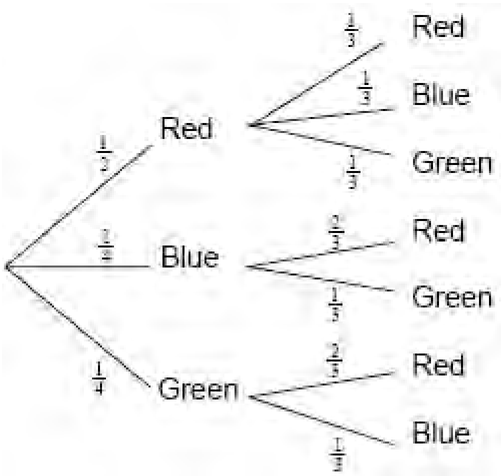
So yes they are statistically independent A1cso 3

<u>Note</u>

1st M1 for attempting all the required probabilities for a suitable test
2nd M1 for use of a correct test – must have attempted all the correct probabilities.
Equality can be implied in line 2.
A1 for fully correct test carried out with a comment

[10]

3. (a)



M1A1A1 3

Note

M1 for shape and labels: 3 branches followed by 3,2,2 with some R, B and G seen

Allow 3 branches followed by 3, 3, 3 if 0 probabilities are seen implying that 3, 2, 2 intended

Allow blank branches if the other probabilities imply probability on blanks is zero

Ignore further sets of branches

1st A1 for correct probabilities and correct labels on 1st set of branches.

2nd A1 for correct probabilities and correct labels on 2nd set of branches.

(accept 0.33, 0.67 etc or better here)

Special Case

With Replacement (This oversimplifies so do not apply Mis -Read: max mark 2/5)

B1 for 3 branches followed by 3, 3, 3 with correct labels and probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ on each.

(b) P(Blue bead and a green bead) = $\left(\frac{1}{4} \times \frac{1}{3}\right) + \left(\frac{1}{4} \times \frac{1}{3}\right) = \frac{1}{6}$

(or any exact equivalent) M1 A1 2

Note

M1 for identifying the 2 cases BG and GB and adding 2 products of probabilities.

These cases may be identified by their probabilities e.g. $\left(\frac{1}{4} \times \frac{1}{3}\right) + \left(\frac{1}{4} \times \frac{1}{3}\right)$

NB $\frac{1}{6}$ (or exact equivalent) with no working scores 2/2

Special Case

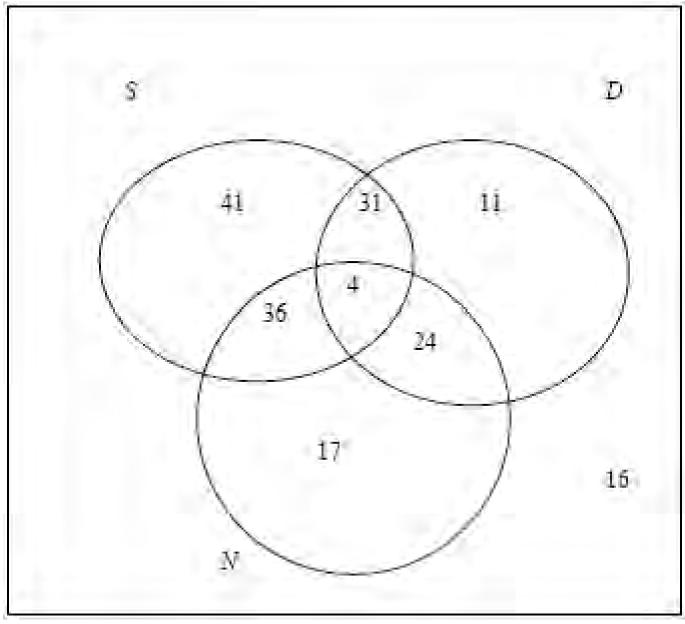
M1 for identifying 2, possibly correct cases and adding 2 products of probabilities but A0 for wrong answer

$\left[\left(\frac{1}{4}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{4}\right)\right]$ will be sufficient for M1A0 here

but $\frac{1}{4}\times\frac{1}{2}+...$ would score M0

[5]

4. (a)



3 closed curves and 4 in centre	M1	
Evidence of subtraction	M1	
31,36,24	A1	
41,17,11	A1	
Labels on loops, 16 and box	B1	5

Note

2nd M1 There may be evidence of subtraction in "outer" portions, so with 4 in the centre then 35, 40 28 (instead of 31,36,24) along with 33, 9, 3 can score this mark but A0A0

N.B. This is a common error and their "16" becomes 28 but still scores B0 in part (a)

(b) P(None of the 3 options) = $\frac{16}{180} = \frac{4}{45}$ B1ft 1

<u>Note</u>

B1ft for $\frac{16}{180}$ or any exact equivalent. Can ft their “16” from their box. If there is no value for their “16” in the box only allow this mark if they have <u>shown</u> some working.

(c) P(Networking only) = $\frac{17}{180}$ B1ft 1

<u>Note</u>

B1ft ft their “17”. Accept any exact equivalent

(d) P(All 3 options/technician) = $\frac{4}{40} = \frac{1}{10}$ M1 A1 2

<u>Note</u>

If a probability greater than 1 is found in part (d) score M0A0

M1 for clear sight of $\frac{\mathrm{P}(S \cap D \cap N)}{\mathrm{P}(S \cap N)}$ and an attempt at one of the probabilities, ft their values.

Allow P(all 3 | $S \cap N$) = $\frac{4}{36}$ or $\frac{1}{9}$ to score M1 A0.

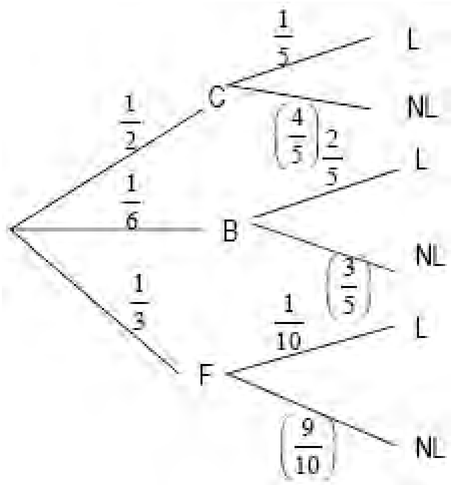
Allow a correct ft from their diagram to score M1A0 e.g. in 33,3,9 case in (a): $\frac{4}{44}$ or $\frac{1}{11}$ is M1A0 A ratio of probabilities with a <u>product</u> of probabilities on top is M0, even with a correct formula.

A1 for $\frac{4}{40}$ or $\frac{1}{10}$ or an exact equivalent

Allow $\frac{4}{40}$ or $\frac{1}{10}$ to score both marks if this follows from their diagram, otherwise some explanation (method) is required.

[9]

5. (a)



Correct tree	B1	
All labels	B1	
Probabilities on correct branches	B1	3

Note

Exact decimal equivalents required throughout if fractions not used e.g. 2(b)(i) 0.03& Correct path through their tree given in their probabilities award Ms

All branches required for first B1. Labels can be words rather than symbols for second B1. Probabilities from question enough for third B1 i.e. bracketed probabilities not required. Probabilities and labels swapped i.e. labels on branches and probabilities at end can be awarded the marks if correct.

(b) (i) $\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$ or equivalent M1 A1 2

Note

Correct answer only award both marks.

(ii) CNL + BNL + FNL $= \frac{1}{2} \times \frac{4}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{9}{10}$ M1

$= \frac{4}{5}$ or equivalent A1 2

Note

At least one correct path identified and attempt at adding all three multiplied pairs award M1

(c) $\mathrm{P}(F'/L) = \dfrac{\mathrm{P}(F' \cap L)}{\mathrm{P}(L)}$ Attempt correct conditional probability **but see notes** M1

$= \dfrac{\frac{1}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{5}}{1-(ii)}$ $\dfrac{\text{numerator}}{\text{denominator}}$ $\dfrac{\text{A1}}{\text{A1ft}}$

$= \dfrac{\frac{5}{30}}{\frac{1}{5}} = \dfrac{5}{6}$ or equivalent cao A1 4

Note

Require probability on numerator and division by probability for M1.Require numerator correct for their tree for M1.

Correct formula seen and used, accept denominator as attempt and award M1

No formula, denominator must be correct for their tree or 1-(ii) for M1

1/30 on numerator only is M0, P(L/F') is M0.

[11]

6. (a) (i) P($A \cup B$) = $a + b$ cao B1

<u>Note</u>

Accept $a + b - 0$ for B1

(ii) P($A \cup B$) = $a + b - ab$ or equivalent B1 2

<u>Special Case</u>

If answers to (i) and (ii) are

(i) P(A)+P(B) and (ii) P(A)+P(B) -P(A)P(B)

award B0B1

7(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded.

(b) P ($R \cup Q$) = 0.15 + 0.35

= 0.5 0.5 B1 1

(c)	$P(R \cap Q) = P(R\|Q) \times P(Q)$				
	$= 0.1 \times 0.35$			M1	
	$= 0.035$		**0.035**	A1	2
(d)	$P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$ OR	$P(R) = P(R \cap Q') + P(R \cap Q)$		M1	
		$= 0.15 +$ their (c)			
	$0.5 = P(R) + 0.35 - 0.035$	$= 0.15 + 0.035$			
	$P(R) = 0.185$	$= 0.185$	0.185	A1	2

[7]

7. (a) E = take regular exercise B = always eat breakfast

$P(E \cap B) = P(E\|B) \times P(B)$	M1	
$= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$	A1	2

__**Note**__

M1 for $\frac{9}{25} \times \frac{2}{3}$ or $P(E \mid B) \times P(B)$ <u>and</u> at least one correct value seen.
A1 for 0.24 or exact equiv.
NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks.

__**Common Errors**__

$\frac{9}{25}$ is M0A0

(b)

$P(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25}$	or $P(E' \mid B')$	or $P(B' \cap E)$	or $P(B \cap E')$	M1	
$= \frac{62}{75}$	$= \frac{13}{25}$	$= \frac{12}{75}$	$= \frac{32}{75}$	A1	
$P(E' \cap B') = 1 - P(E \cup B) = \frac{13}{75}$ or $0.17\dot{3}$				M1 A1	4

Note

1st M1 for use of the addition rule. Must have 3 terms and some values, can ft their (a) Or a full method for P(E'|B') requires 1 – P(E|B') and equation for P(E|B'): (a) + $\frac{x}{3}=\frac{2}{5}$ Or a full method for P($B' \cap E$) or P($B \cap E'$) [or other valid method]

2nd M1 for a method leading to answer e.g. 1 – P($E \cup B$) or P(B') × P($E'|B'$)or P(B') – P($B' \cap E$) or P(E') – P($B \cap E'$)

Venn Diagram 1st M1 for diagram with attempt at $\frac{2}{5} - \mathrm{P}(B' \cap E)$ or $\frac{2}{3} - \mathrm{P}(B \cap E)$. Can ft their (a)

1st A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram

2nd M1 for attempting 75 – their (18 +32 + 12)

(c) P(E | B) = 0.36 ≠ 40 = P(E) or P($E \cap B'$) = $\frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = \mathrm{P}(E) \times \mathrm{P}(B)$ M1

A1 2

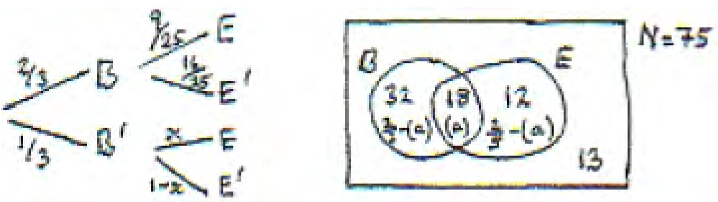
So E and B are not statistically independent

Note

M1 for identifying suitable values to test for independence e.g. P(E) = 0.40 and P(E|B) = 0.36
Or P(E) × P(B) = ... and P($E \cap B$) = their (a) [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$].
Values seen somewhere

A1 for correct values and a correct comment

Diagrams You may see these or find these useful for identifying probabilities.



Common Errors

(a) $\frac{9}{25}$ is M0A0

(b) P(EUB) = $\frac{53}{75}$ scores M1A0 1– P($E \cup$ B) = $\frac{22}{75}$ scores M1A0

(b) P(B') × P(E') = $\frac{1}{3} \times \frac{3}{5}$ scores 0/4

[8]

8.	(a)	$E(X) = 0 \times 0.4 + 1 \times 0.3 + ... + 3 \times 0.1, = 1$	M1, A1	2

Note

M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by $k(\neq 1)$ is M0.

(b)	$F(1.5) = [P(X \le 1.5) =]\ P(X \le 1),\ = 0.4 + 0.3 = 0.7$	M1, A1	2

Note

M1 for $F(1.5) = P(X \le 1)$.[**Beware**: $2 \times 0.2 + 3 \times 0.1 = 0.7$ but scores M0A0]

(c)	$E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + ... + 3^2 \times 0.1, = 2$	M1, A1	
	$Var(X) = 2 - 1^2, = 1$ (*)	M1, A1cso	4

Note

1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0. Condone calling $E(X^2) = Var(X)$.

ALT

1st A1 is for an answer of 2 or a fully correct expression.

2nd M1 for $-\mu^2$, condone $2 - 1$, unless clearly $2 - \mu$ Allow $2 - \mu^2$ with $\mu = 1$ even if $E(X) \neq 1$

2nd A1 for a fully correct solution with no incorrect working seen, **both** Ms required.

$\underline{\Sigma\ (x - \mu)^2 \times P(X = x)}$

1st M1 for an attempt at a full list of $(x - \mu)^2$ values and probabilities. 1st A1 if all correct

2nd M1 for at least 2 non-zero terms of $(x - \mu)^2 \times P(X = x)$ seen. 2nd A1 for $0.4 + 0.2 + 0.4 = 1$

(d)	$Var(5 - 3X) = (-3)^2\ Var(X),\ = 9$	M1, A1	2

Note

M1 for use of the correct formula. $-3^2\ Var(X)$ is M0 unless the final answer is >0.

(e)

Total	Cases	Probability	
	$(X = 3) \cap (X = 1)$	$0.1 \times 0.3 = 0.03$	
4	$(X = 1) \cap (X = 3)$	$0.3 \times 0.1 = 0.03$	
	$(X = 2) \cap (X = 2)$	$0.2 \times 0.2 = 0.04$	B1B1B1
	$(X = 3) \cap (X = 2)$	$0.1 \times 0.2 = 0.02$	
5	$(X = 2) \cap (X = 3)$	$0.2 \times 0.1 = 0.02$	M1
6	$(X = 3) \cap (X = 3)$	$0.1 \times 0.1 = 0.01$	A1

Total probability = 0.03 + 0.03 + 0.04 +0.02 + 0.02 + 0.01 = 0.15 A1 6

Note

Can follow through their Var(X) for M1

ALT

1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0
2nd B1 for all cases listed for 2 totals }
3rd B1 for a complete list of all 6 cases } These may be highlighted in a table

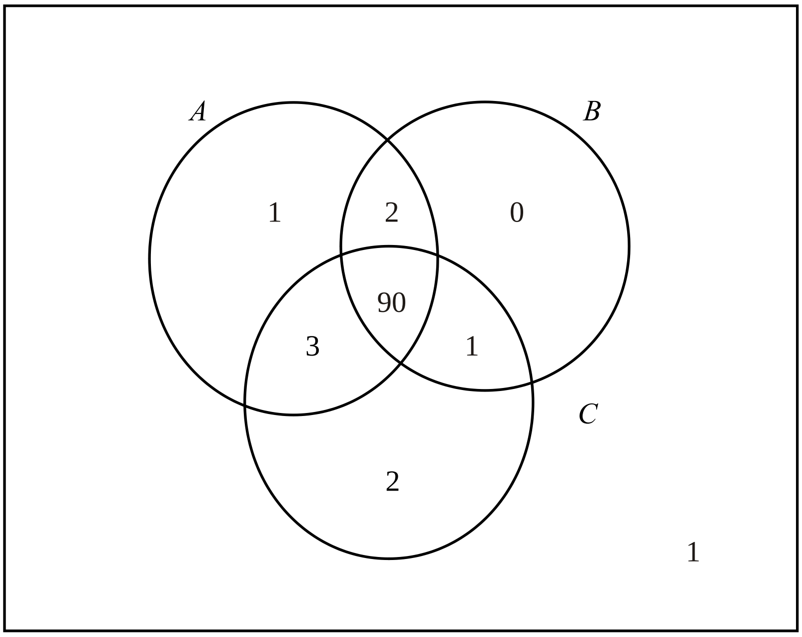
Using Cumulative probabilities

1st B1 for one or more cumulative probabilities used e.g.2 then 2 or more or 3 then 1 or more
2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥2; 3, ≥1
M1 for one correct pair of correct probabilities multiplied
1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.
2nd A1 for 0.15 or exact equivalent only as the final answer.

[16]

9. **Diagram may be drawn with $B \subset (A \cup C)$ or with the 0 for $B \cap (A \cup C)'$ simply left blank**

(a)



3cc	M1	
90, 3, 2, 1	A1	
1, (0), 2	M1A1	
1 outside	A1	
Box	B1	6

Accept decimals or probs. in Venn diagram

1st M1 for 3 closed, labelled curves that overlap.
A1 for the 90, 3, 2 and 1

2nd M1 for one of 1, 0 or 2 correct or a correct sum of 4 values for A, B or C

2nd A1 for all 7 values correct. Accept a blank instead of 0.

NB final mark is a B1 for the box not an A mark as on EPEN

(b) P(none) = 0.01 B1ft 1

B1ft Follow through their '1'from outside divided by 100

(c) P(A but not B) = 0.04 M1 A1ft 2

M1 for correct expression eg $P(A \cup B) - P(B)$ or calculation e.g. 3 + 1 or 4 on top

A1 for a correct probability, follow through with their '3 + 1' from diagram

(d) P(any wine but C) = 0.03 M1A1ft 2

M1 for correct expression or calculation e.g. 1 + 2 + 0 or 99 – 96 or 3 on top

A1 for a correct probability, follow through their '2 + 1 + 0' from diagram

(e) P(exactly two) = 0.06 M1A1ft 2

M1 for a correct expression or calculation e.g. 3 + 2 + 1 or 6 on top

(f) $P(C \mid A) = \dfrac{\text{P}(C \cap A)}{\text{P}(A)} =, \dfrac{93}{96}$ or $\dfrac{31}{32}$ or AWRT 0.969 M1A1ft,A1 3

M1 for a correct expression upto "," and some correct substitution, ft their values.
One of these probabilities must be correct or correct ft.
If P(C) on bottom M0

1st A1ft follow through their $A \cap C$ and their A but the ratio must be in (0, 1)

2nd A1 for correct answer only.
Answer only scores 3/3, but check working
P($A \cap C$)/P(C) is M0

In parts (b) to (f) full marks can be scored for correct answers or correct ft

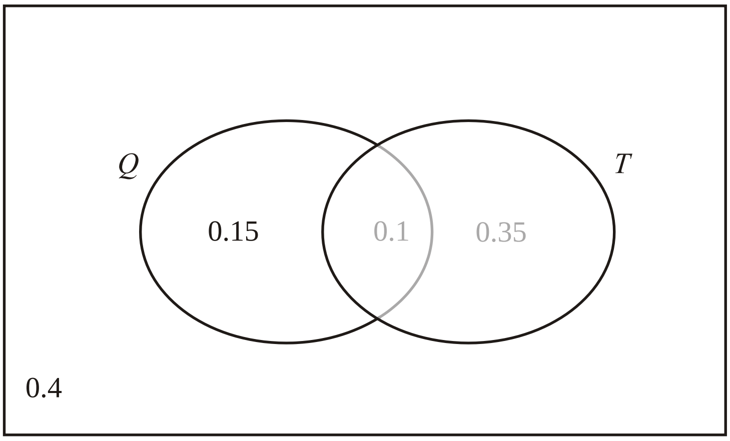
For M marks in (c) to (e) they must have a fraction

[16]

10. (a) P($Q \cup T$) = 0.6 B1
P(Q) + P(T) – P($Q \cap T$) = 0.6 M1
P($Q \cap T$) = 0.1 A1 3

B1 for 0.6
M1 for use of P(Q) + P(T) – P($Q \cap T$) = P($Q \cup T$)
0.1 Correct answer only for A1
Alternative:
(25 + 45 + 40 =) 110% B1
110 – 100 = 10% M1A1
0.1 stated clearly as the final answer with no working gets 3/3

(b)



Venn	M1	
0.15, 0.35	A1ft	
0.4 and box	B1	3

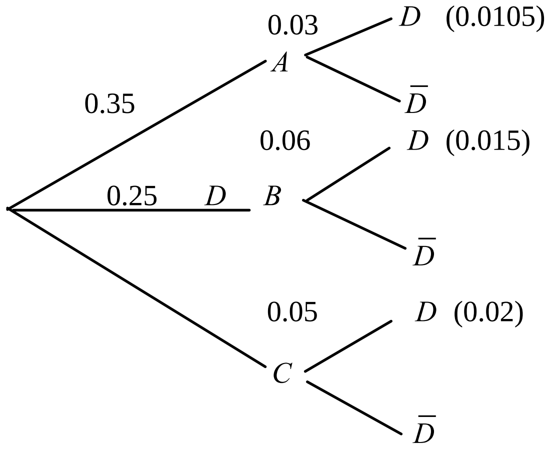
Two intersecting closed curves for M1, no box required.
At least one label (Q or T) required for first A1.
Follow through (0.25 – 'their 0.1') and (0.45 – 'their 0.1') for A1.
0.4 and box required, correct answer only for B1
Using %, whole numbers in Venn diagram without % sign,
whole numbers in correct ratio all OK

(c) $\mathrm{P}(Q \cap T' \mid Q \cup T) = \dfrac{0.15}{0.60} = \dfrac{1}{4}$ or 0.25 or 25% M1A1ftA1 3

require fraction with denominator 0.6 or their equivalent from
Venn diagram for M1
Follow through their values in fraction for A1
Final A1 is correct answer only.
<u>No working no marks.</u>

[9]

11. (a)



Correct tree shape	M1	
A, B and *C* and 0.35 and 0.25	A1	
D (x3) and 0.03, 0.06, 0.05 (May be implied by seeing P($A \cap D$) etc at the ends)	A1	3

M1 for tree diagram, 3 branches and then two from each.
At least one probability attempted.

(b) (i) $P(A \cap D) = 0.35 \times 0.03$, = $\underline{\mathbf{0.0105}}$ or $\frac{21}{2000}$ M1, A1

P(C) = 0.4 (anywhere) B1

(ii) P(D) = (i) + 0.25 × 0.06 + (0.4 × 0.05) M1

= $\underline{\mathbf{0.0455}}$ or $\frac{91}{2000}$ A1 5

1st M1 for 0.35 × 0.03. Allow for equivalent from <u>their</u> tree diagram,
B1 for P(C) = 0.4, can be in correct place on tree diagram or implied by 0.4 × 0.05 in P(D).
2nd M1 for all 3 cases attempted and <u>some</u> correct probabilities seen, including +. Can ft their tree.

Condone poor use of notation if correct calculations seen.
E.g. P(C | D) for P($C \cap D$).

(c) $\text{P}(C|D) = \dfrac{\text{P}(C \cap D)}{\text{P}(D)}, = \dfrac{0.4 \times 0.05}{\text{(ii)}}$ M1, A1ft

$= 0.43956...$ or $\dfrac{40}{91}$ **<u>0.44</u>** or awrt **<u>0.440</u>** A1 3

[Correct answers only score full marks in each part]

[11]

M1 for attempting correct ratio of probabilities.
There must be an attempt to substitute some values in a correct formula.
If no correct formula and ration not correct ft score M0.
Writing P(D|C) and attempting to find this is M0.
Writing P(D|C) but calculating correct ratio – ignore notation and mark ratios.

A1ft must have their 0.4 × 0.05 divided by their (ii).

If ratio is incorrect ft (0/3) unless correct formula seen and part of ratio is correct then M1.

12. (a) N.B. Part (a) doesn't have to be in a table, could be a list P(X = 1) = …etc B1, B1, B1 3

x	1	2	3	4	5	6
P(X = x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

0.0278, 0.0833, 0.139, 0.194, 0.25, 0.306 (Accept awrt 3 s.f)

1st B1 for x = 1, … 6 and at least one correct probability
N.B. $\frac{3}{36} = \frac{1}{12}$ and $\frac{9}{36} = \frac{1}{4}$

2nd B1 for at least 3 correct probabilities

3rd B1 for a fully correct probability distribution.

(b) P(3) + P(4) + P(5) =, $\underline{\dfrac{21}{36} \text{ or } \dfrac{7}{12}}$ <u>or awrt 0.583</u> M1, A1 2

M1 for attempt to add the correct three probabilities, ft their probability distribution

(c) E(X) = $\frac{1}{36}+2\times\frac{3}{36}+\ldots, = \underline{\frac{161}{36}}$ or $4.47\dot{2}$ or $4\frac{17}{36}$ M1, A1 2

M1 for a correct attempt at $E(X)$. Minimum is as printed.
Exact answer only scores M1A1.

[Division by 6 at any point scores M0, no ISW. Non-exact answers with no working score M0.]

(d) E(X^2) = $\frac{1}{36}+2^2\times\frac{3}{36}+\ldots, = \frac{791}{36}$ or full expression or $21\frac{35}{36}$
or awrt 21.97 M1, A1

Var(X) = $\frac{791}{36}-\left(\frac{161}{36}\right)^2$, = **<u>1.9714… *</u>** M1, A1cso 4

1st M1 for a correct attempt at $E(X^2)$.
Minimum as printed.
$\frac{791}{36}$ or awrt 21.97 scores M1A1.

2nd M1 for their $E(X^2)$ – (their E(X))^2.

2nd A1 cso needs awrt 1.97 <u>and</u> $\frac{791}{35}-\left(\frac{161}{36}\right)^2$ or $\frac{2555}{1296}$ or any fully correct expression seen.

Can accept <u>at least 4 sf</u> for both. i.e. 21.97 for $\frac{791}{36}$,
4.472 for $\frac{161}{36}$, 20.00 for $\left(\frac{161}{36}\right)^2$.

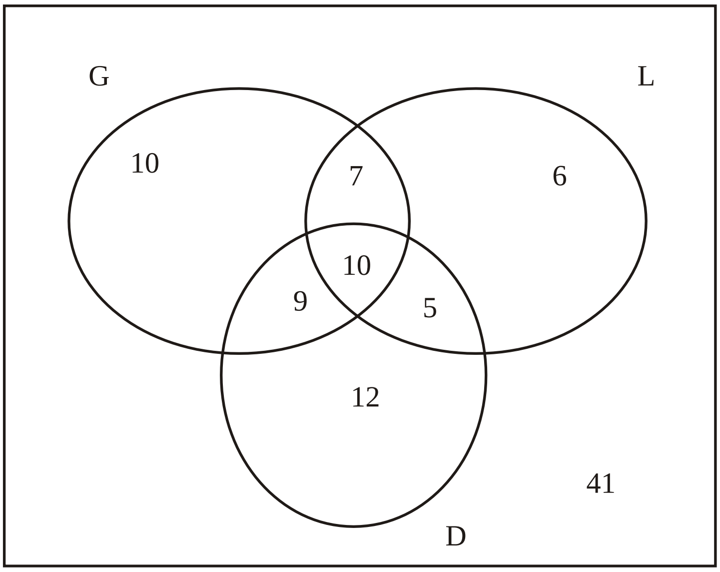
(e) Var(2 – 3X) = 9 × 1.97 or (–3)^2 × 1.97, = 17.73 awrt **<u>17.7</u>** or $\frac{2555}{144}$ M1, A1 2

M1 for correct use of Var($aX + b$) formula or a <u>full</u> method.

NB –3^2 × 1.97 followed by awrt 17.7 scores M1A1
<u>BUT</u> –3^2 × 1.97 alone, or followed by – 17.7, scores M0A0.

[13]

13. (a)

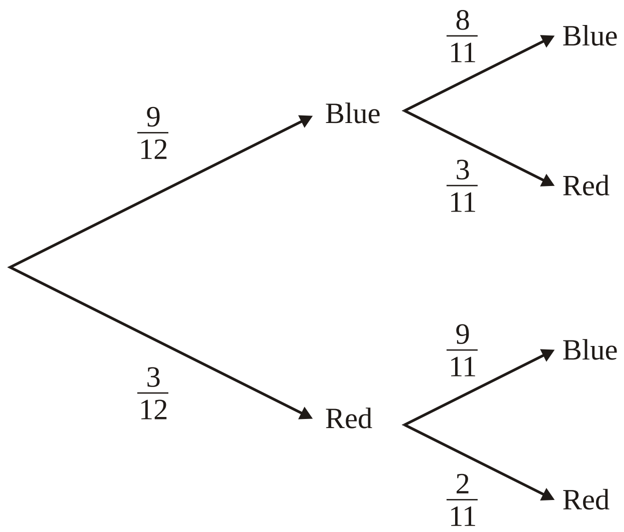


	3 closed curves that intersect	M1	
	Subtract at either stage	M1	
	9, 7, 5	A1	
	10, 6, 12	A1	
	41 & box	A1	6
(b)	$\mathrm{P}(\mathrm{G}, \overline{\mathrm{LH}}, \overline{\mathrm{D}}) = \frac{10}{100} = \frac{1}{10}$	B1ft	1
(c)	$\mathrm{P}(\mathrm{G}, \overline{\mathrm{LH}}, \overline{\mathrm{D}}) = \frac{41}{100}$	B1ft	1
(d)	P(only two attributes) $-\frac{9+7+5}{100} = \frac{21}{100}$	M1 A1	2
(e)	P(G $\mid$ LH&DH) = $\frac{\mathrm{P}(\mathrm{G}\ \&\ \mathrm{LH}\ \&\ \mathrm{DH})}{\mathrm{P}(\mathrm{LH}\ \&\ \mathrm{DH})} = \frac{\frac{10}{100}}{\frac{15}{100}} = \frac{10}{15} = \frac{2}{3}$ awrt 0.667	M1 A1ft A1	3

N.B. Assumption of independence M0

[13]

14. (a)



Tree	M1	
$\frac{9}{12}, \frac{3}{12}$	A1	
Complete & labels	A1	3

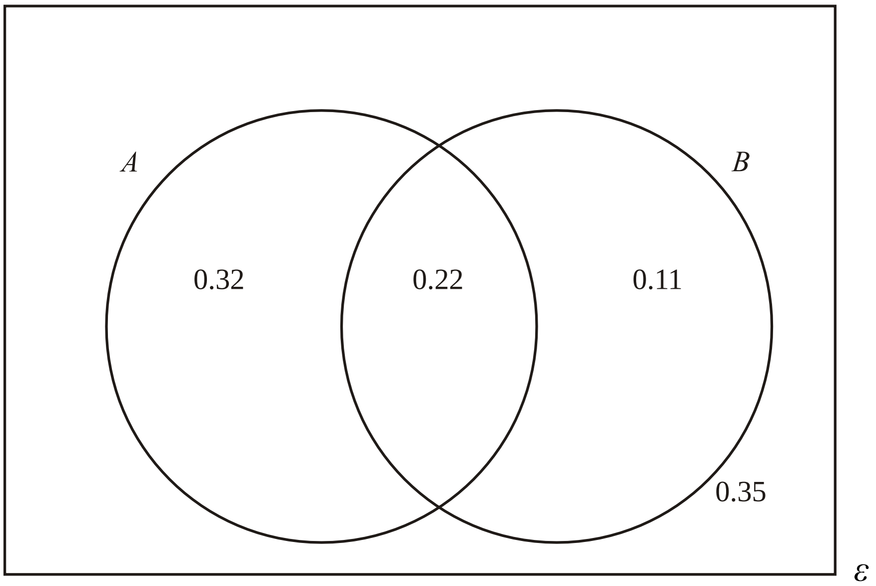
(b) P(Second ball is red) = $\frac{9}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{1}{4}$ M1 A1 2

(c) P(Both are red | Second ball is red) = $\dfrac{\frac{3}{12} \times \frac{2}{11}}{\frac{1}{4}} = \frac{2}{11}$

exact or awrt 0.182 M1 A1 2

[7]

15. (a)



Venn Diagram	M1	
0.32, 0.11 & A, B	A1	
0.22, 0.35 & box	A1	3

(b) P(A) = 0.32 + 0.22 = 0.54; P(B) = 0.33 M1A1ft, A1ft 3

(c) $P(A|B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{32}{67}$ awrt 0.478 M1 A1 2

(d) For independence $P(A \cap B') = P(A)P(B)$ M1 A1ft

For these data $0.22 \neq 0.54 \times 0.33 = 0.1782$

(OR $P(A|B') \neq P(A)$ for M1A1ft

OR $\dfrac{2}{3} = P(A|B) \neq P(A) = 0.54$ for M1A1ft A1ft 3

$\therefore$ NOT independent

[11]

16.

	Glasses	No Glasses	Totals		
Science	18	**12**	30		
Arts	**27**	**23**	**50**	50 may be seen in (a)	
Humanities	44	**24**	68	23 may be seen in (b)	B1
					B1
Totals	89	**59**	148		

(a) P(Arts) = $\frac{50}{148} = \frac{25}{74}$ = 0.338 M1 A1 4

a number/148

(b) P(No glasses / Arts) = $\frac{23/148}{50/148} = \frac{23}{50}$ = 0.46 M1 A1 2

$\frac{\textit{prob}}{\textit{their}(a)\,\textit{prob}}$ *or* $\frac{\textit{number}}{\textit{their 50}}$

(c) P(Right Handed) = $\left(\frac{30}{148} \times 0.8\right) + \left(\frac{50}{148} \times 0.7\right) + \left(\frac{68}{148} \times 0.75\right)$ M1 A1ft

attempt add three prob
A1 ft on their (a)

= $\frac{55}{74}$ = 0.743 A1 3

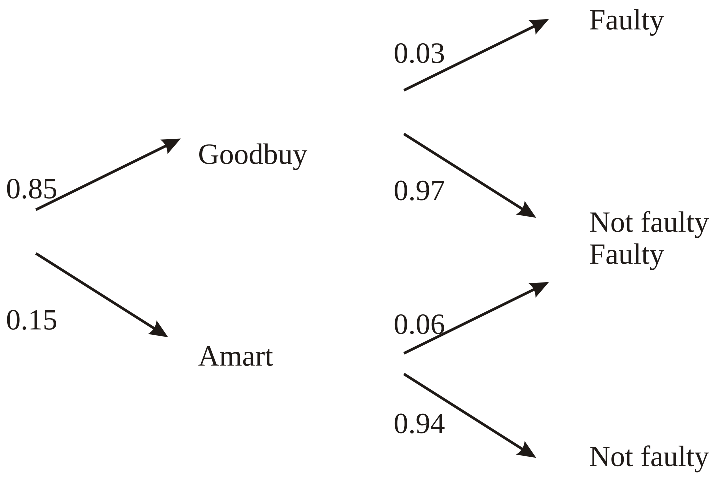
awrt 0.743

(d) P(Science /Right handed0 = $\frac{\frac{30}{148} \times 0.8}{(c)} = \frac{12}{55}$ = 0.218 M1 A1ft A1 3

ft on their (c)

[12]

17.



Tree (both sections)	M1	
labels & 0.85, 0.15 or equiv.	A1	
0.03,0.97,0.06,0.94	A1	3

(b) P(Not faulty) = $(0.85 \times 0.97) + (0.15 \times 0.94)$ M1,A1∫

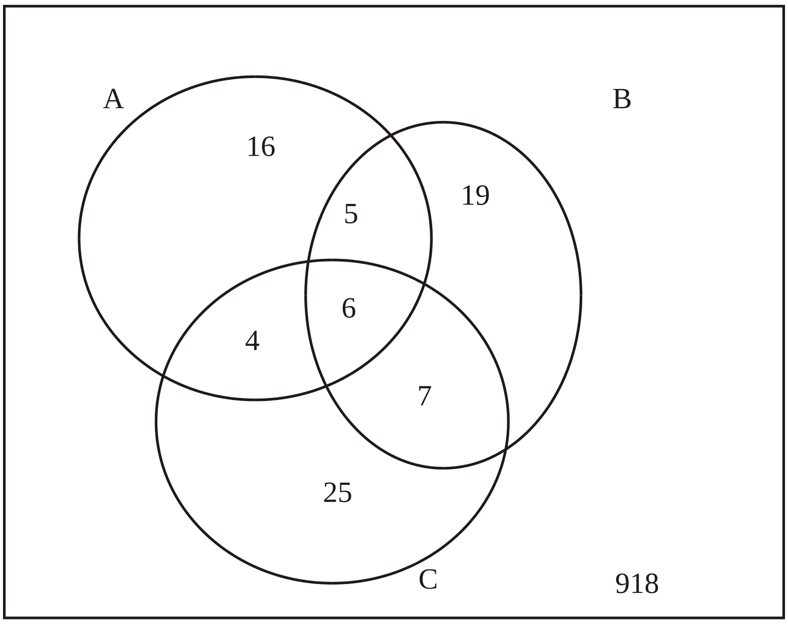
valid path & their values, correct

0.9655 A1 3

% or 1931/2000 or equiv. or awrt 0.966

[6]

18. (a)



6	B1	
subtract	M1	
4,5,7	A1	
subtract	M1	
16,19,25	A1	
box & 918	B1	6

(b) P(No defects) = $\dfrac{918}{1000}$ = 0.918 B1ft 1

(c) P(No more than 1) = $\dfrac{918+16+19+25}{1000}$ **OR** $1 - \dfrac{5+6+4+7}{1000}$ M1

=0.978 A1ft 2

0.978

(d) P(B| Only 1 defect) = $\frac{\text{P(B and 1 defect)}}{\text{P(1 defect)}} = \frac{\frac{19}{1000}}{\frac{16+19+25}{1000}}$ M1

conditional prob

$= \frac{19}{60}$ A1ft 2

$\frac{19}{60}$ *or* $0.31\dot{6}$ *or* 0.317

(e) P(Both had type B) = $\frac{37}{1000} \times \frac{36}{999}$ M1

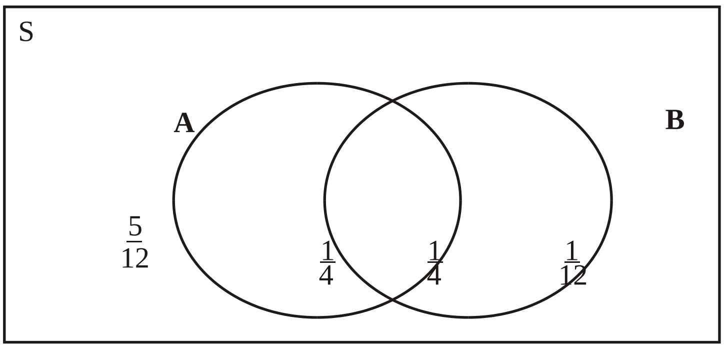
theirs from B ×

$= \frac{37}{27750}$ or $0.001\dot{3}$ or 0.00133 or equivalent A1 2

cao

[13]

19. (a)



2 intersecting closed curves in a box M1

both $\frac{1}{4}, \frac{1}{12}$ B1,B1

$\frac{5}{12}$ B1ft 4

(b) P($A \cup B$) = $\frac{7}{12}$ B1ft 1

0.583 or $0.58\dot{3}$ *or* $\frac{7}{12}$

(c) $P(A|B) = \dfrac{P(A \cup B')}{P(B')} = \dfrac{\frac{1}{4}}{\frac{2}{3}} = \dfrac{3}{8}$ or 0.375 M1,A1 2

their fractions divided, cao

[7]

20.

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

$2 \times (1, 2, \ldots, 3)$ M1

Adding M1

All ≥ 5 correctly indicated A1

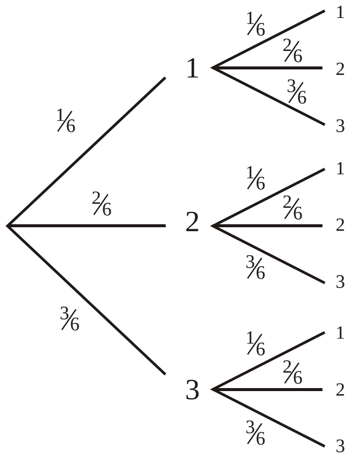
$\therefore$ P (sum at least 5) = $\dfrac{21}{36} = \dfrac{7}{12}$

Attempt to count ≥ 5 M1

$\dfrac{21}{36}$; $\dfrac{7}{12}$; $0.58\dot{3}$; *0.583* A1 5

[5]

Alt 1



Tree with relevant branches	M1	
All correct – $\frac{2}{6}$, $\frac{3}{6}$ on those branches	A1	
P(sum ≥ 5) = $\left(\frac{2}{6}\times\frac{3}{6}\right)+\left(\frac{3}{6}\times\frac{2}{6}\right)$ (At least 2 pairs & adding)	M1	
+ $\left(\frac{3}{6}\times\frac{3}{6}\right)$ all correct	A1	
= $\frac{21}{36}$; $\frac{7}{12}$; $0.58\dot{3}$; 0.583	A1	5

Alt 2

Outcomes (2, 3), (3, 3), (3, 2)

Recognising 2 pairs *Can be implied*	M1	
All correct	A1	

P(sum ≥ 5) = $\left(\frac{2}{6}\times\frac{3}{6}\right)+\left(\frac{3}{6}\times\frac{3}{6}\right)+\left(\frac{3}{6}\times\frac{2}{6}\right)$

Multiplying 2 pairs of 2 probs. & adding	M1	
All correct	A1	
= $\frac{21}{36}$	A1	5

Alt 3

P (sum ≥ 5) = 12 $\left(\frac{1}{6}\times\frac{1}{6}\right)$ + 9 $\left(\frac{1}{6}\times\frac{1}{6}\right)$

$a(p_1 \times p_2)$ or $b(p_1 \times p_2)$	M1	
$p_1 = p_2 = \frac{1}{6}$	A1	
$a(\ ) + b(\ )$	M1	

$= \underline{\frac{21}{36}}$

21 or 12 + 9	A1	
$\frac{21}{36}$; $\frac{7}{12}$ etc	A1	5

Alt 4

x	2	3	4	5	6
P($X = x$)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

2, 3, 4, 5, 6	M1
Adding probabilities	M1
All correct	A1

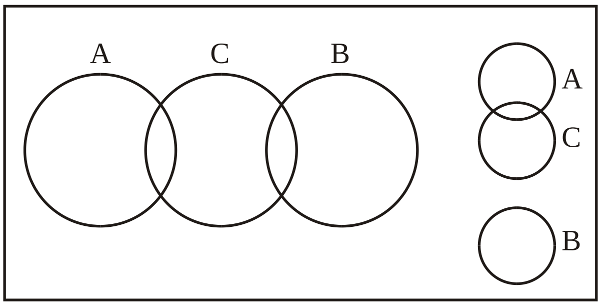
$\therefore$ P($X \geq 5$) = $\frac{12}{36} + \frac{9}{36}$

Adding P(5) & P(6)	M1

$= \underline{\frac{21}{36}}$

$\frac{21}{36}$; $\frac{7}{12}$ etc	A1	5

21. (a)



A, B, C inside S	B1	
A, B no overlap	B1	
A, C overlap	B1	3

(b) $\mathrm{P}(A|C) = \dfrac{\mathrm{P}(A \cap C)}{\mathrm{P}(C)} = \dfrac{\mathrm{P}(A)\,\mathrm{P}(C)}{\mathrm{P}(C)} = \mathrm{P}(a)$ M1

Use of independence

$= \underline{0.2}$ A1 2

(c) $\mathrm{P}(A \cup B) = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P}(A \cap B)$ M1

use of $P(A \cup B)$ & $P(A \cap B) = 0$ can be implied

$= 0.2 + 0.4 - 0$

$= \underline{0.6}$ A1 2

SR: No working B1 only

(d) $\mathrm{P}(A \cup C) = \mathrm{P(A)} + \mathrm{P(C)} - \mathrm{P}(A \cap C)$ M1

Use of $P(A \cup C)$ & independence

$\therefore 0.7 = 0.2 + \mathrm{P(c)} - 0.2\,\mathrm{P(C)}$ A1

$\therefore 0.5 = \mathrm{P(C)}\,\{1 - 0.2\}$ M1

Solving for P(C) from an equation with 2P(C) terms

$\therefore \underline{\mathrm{P(c)} =}\ \frac{5}{8}$; 0.625 A1 4

[11]

<u>NB</u> $P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= 0.4 + 0.625 - P(B \cap C) \Rightarrow P(B \cap C) > 0$

22. (a) (i) $P(A \cap B') = P(A/B') = \frac{4}{5} \times \frac{1}{2} = \frac{4}{10} = \underline{\frac{2}{5}}$ M1

Use of P(A/B′)P(B′) A1

(ii) $P(A \cap B) = P(A) - P(A \cap B')$ M1

$= \frac{2}{5} - \frac{2}{5}$

$= \underline{0}$ A1

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$= \frac{2}{5} + \frac{1}{2} - 0$

$= \underline{\frac{9}{10}}$ A1ft

(iv) $P(A/B) = P\frac{(A \cap B)}{P(B)} = 0$ B1 7

(b) (i) since $P(A \cap B) = 0$ seen B1

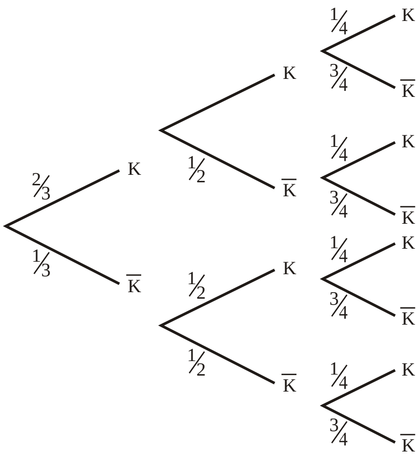
A and B are mutually exclusive B1 2

(ii) Since P (A/B) ≠ P (a) or equivalent B1

A and B are NOT independent B1 2

[11]

23. (a)



Tree with correct number of branches	M1	
$\frac{2}{3}, \frac{1}{3}$	A1	
$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	A1	
$\frac{1}{4}, \frac{3}{4} \ldots \frac{3}{4}$	A1	4

(b) P (All 3 Keys) = $\frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{2}{24} = \frac{1}{12}$ M1 A1 2

$\frac{1}{12}$; 0.08$\dot{3}$; 0.0833

(c) P(exactly 1 key) = $\left(\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right)$ 3 triples added M1

$= \underline{\frac{10}{24} = \frac{5}{12}}$

Each correct

$\frac{10}{24}$; $\frac{5}{12}$; 0.41$\dot{6}$; 0.417 A1 A1 A1 A1 5

(d) P (Keys not collected on at least 2 successive stages)

$$=\left(\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{3}{4}\right)$$

3 triples added M1
Each correct A1 A1 A1

$=\frac{10}{24}=\frac{5}{12}$ A1 5

$\frac{10}{24}$; $\frac{5}{12}$; $0.41\dot{6}$; 0.417

Alternative:
1 – P (Keys collected on at least 2 successive stages) M1

$= 1 - \left\{\left(\frac{2}{3}\times\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}\right)\right\}$ A1 A1 A1

$= \frac{5}{8}$ A1 5

[16]

24. (a) P(scores 30 points) = P(hit, hit, hit,) = $0.6^3 = 0.216 = \frac{27}{125}$ 0.6^3 M1

$\frac{27}{125}$; 0.216 A1 2

(b)

x	0	10	20	30
	0.4	0.6 × 0.4	$0.6^2 \times 0.4$	
P ($X = x$)	0.4	0.24	0.144	(0.216)
	$\frac{4}{10}$	$\frac{6}{25}$	$\frac{18}{225}$	

x = 0, 10, 20, 30 B1
One correct
P($X = x$) M1
0.4; 0.24; 0.144 A1; A1; A1 5

(c)	$E(X) = (0 \times 0.4) + \ldots + (30 \times 0.216) = \underline{11.76}$			
	$\Sigma xP(X = x)$ *Their distribution* M1 *AWRT 11.8* A1			
	$E(X^2) = (10^2 \times 0.24) + \ldots + (30^2 \times 0.216) = 276$	B1		
	Std Dev = $\sqrt{276 - 11.76^2} = 11.7346\ldots$ $\sqrt{E(X^2) - (E(X))^2}$	M1		
	3 s.f. 11.7	A1	5	
(d)	P (Linda scores more in round 2 than in round 1) $= P(X_1 = 0 \ \& \ X_2 = 10, 20, 30)$ $X_2 > X_1$ $+ P(X_1 = 10 \ \& \ X_2 = 20, 30)$	M1		
	Can be implied			
	All possible	A1		
	$+P(X_1 = 20 \ \& \ X_2 = 30)$	A1		
	$= 0.4 \times (0.24 + 0.144 + 0.216) = 0.24$	A1 ft		
	$+ 0.24 \times (0.144 + 0.216) = 0.0864$	A1		
	$+ (0.144 \times 0.216) = 0.031104$	A1 ft		
	$\underline{= 0.357504}$	A1	6	
	AWRT 0.358			
				[18]

25. (a)	A list of <u>all possible outcomes</u> of an experiment *Accept examples*	B1	1	
(b)	A sub-set of outcomes of an experiment	B1	1	
(c)	$P(A \cap B) = P(A)P(B) = \frac{1}{3} \times \frac{1}{4} = \underline{\frac{1}{12}}$	B1	1	
(d)	$P(A \mid B) = P(A) = \frac{1}{3}$	M1		
	Application of indep.			
	1/3	A1 cao	2	

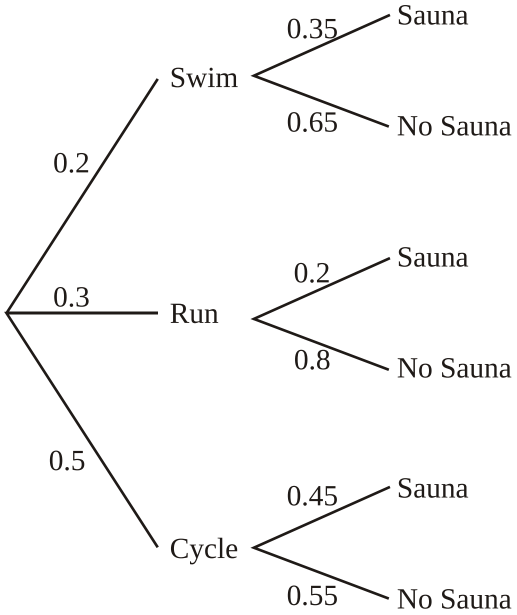
	(e)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$		
		$= \frac{1}{3} + \frac{1}{4} - \frac{1}{12}$	M1	
		Application of $P(A \cup B)$		
		$= \underline{\frac{1}{2}}$	A1 cao	2

[7]

26.	(a)	$P(A \cap B) = \frac{10}{100} = \frac{1}{10} = 0.1$	M1 A1	2
	(b)	$P(A') = \frac{75}{100} = 0.75$	M1 A1	2
	(c)	$P(B' \mid A) = \frac{P(B' \cap A)}{P(A)} = \frac{\frac{15}{100}}{\frac{25}{100}} = \frac{15}{25} = \frac{3}{5} = 0.6$	M1 A1	2
	(d)	$P(A' \cap B) = 0.4$; $P(A')P(B) = 0.75 \times 0.5 = 0.375$	M1	
		Since $P(A' \cap B) \neq P(A')P(B) \Rightarrow$ not independent	A1	
		One of models is less reliable	A1	3

[9]

27. (a)



Tree with correct number of branches	M1	
0.2, 0.3, 0.5	A1	
All correct	A1	3

(b) P(used sauna) = $(0.2 \times 0.35) + (0.3 \times 0.2) + (0.5 \times 0.45)$ M1 A1

$= 0.355$ A1 3

(c) P(swim | sauna used) = $\dfrac{\text{P(swim \& sauna)}}{\text{P(sauna)}}$

$= \dfrac{0.2 \times 0.35}{0.355}$ M1 A1

$= 0.19718$ (accept awrt 0.197) A1 3

(d) P(swim | sauna not used) = $\dfrac{\text{P(sauna not used} \mid \text{swim)P(swim)}}{\text{P(sauna not used)}}$ M1

P(sauna not used | swim) = $1 - 0.35 = 0.65$ B1

P(sauna not used) $= 1 - 0.355 = 0.645$ M1 A1 ft

$\therefore$ P(swim | sauna not used) $= \dfrac{0.65 \times 0.2}{0.645}$ M1

$= 0.20155$

(accept awrt 0.202) A16

[15]

1. Overall there were very few errors made when candidates completed their tree diagrams. A small number of candidates repeated their probabilities of $\frac{2}{3}$ (for obtaining a head) and $\frac{1}{3}$ (for obtaining a tail) on the second branches for the fair coin. Occasionally the $\frac{5}{12}$ and $\frac{7}{12}$ probabilities were placed on the wrong branches and, in a few instances, quantities rather than probabilities were used. The vast majority of candidates were able to calculate the probability that Shivani selects a head correctly, or at least follow through the correct method from their tree diagrams, with few errors seen.

In contrast the quality of candidates' attempts at part (c) was extremely varied. Very few candidates quoted the correct formula despite it being given in the formula booklet, and of those who did, few realised that the numerator should be $\frac{5}{12} \times \frac{2}{3}$. The numerator was quite often seen as $\frac{5}{12}$ alone, and a number of candidates failed to recognise that their denominator should be their answer to part (b), leading in some cases to a repeated fraction in the numerator and denominator. P(H/R) was sometimes calculated instead of P(R/H).

The final part of the question was attempted fairly successfully overall. Indeed, many of the candidates who had erred in previous parts of the question were able to gain some credit, as most could identify at least one of $(\frac{5}{12})^2$ or $(\frac{7}{12})^2$. The special case pertaining to no replacement was occasionally seen.

2. Overall this question proved to be quite challenging for candidates and incorrect interpretation of the Venn diagram lost many candidates marks. In spite of this, most candidates had no trouble proving the given probability in part (a).

In part (b), however, quite a number of candidates neglected one of the four components of the numerator, usually the 3, and $\frac{11}{30}$ was consequently an extremely common wrong answer. Other wrong answers included $\frac{9}{30}$, $\frac{13}{30}$ and $\frac{16}{30}$. Some candidates chose to use the addition rule, which was generally written down correctly, although quite often P(A) was given as $\frac{4}{30}$ and P(B) as $\frac{5}{30}$, giving rise to $P(A \cup B) = \frac{7}{30}$.

In contrast, the majority of candidates were able to deduce that $P(A \cap C) = 0$ and quite a few gave explanations as part of their answer, such as 'there is no overlap', or 'no intersection' and some even discussed the idea of mutual exclusivity. A small proportion of candidates had the right idea but failed to give a probability, giving their answer as 'nobody' or in a few cases 'the empty set'. However, not all of the candidates realised that mutually exclusive events have a probability of 0 of occurring together and some mistakenly thought that $P(A \cap C)$ equalled $P(A)P(C)$ here.

Answers to part (d) were extremely varied. Most candidates did not recognise that a conditional probability was required and consequently did not obtain the correct denominator. Common wrong answers were $\frac{6}{30}$, $\frac{6}{20}$ and $\frac{3}{20}$. A significant number attempted to perform some complex calculations in which they tried unsuccessfully to use the formula for conditional probability. Very few candidates used the Venn diagram to calculate the probability directly.

Testing for independence was generally performed successfully overall, with the majority of candidates carrying out suitable tests. However, some candidates did find this challenging and often the wrong probabilities were compared and some incorrect probabilities were obtained. A number of candidates appeared to be confusing independence with mutual exclusivity. Some candidates merely provided a comment on the perceived nature of independence without performing any calculations at all. Of those candidates who were successful, the most common approach was to test whether $P(B \cap C) = P(B)P(C)$, although there were a few cases where $P(A \cap C)$ was compared with $P(A)P(C)$ by mistake. Rather worryingly, a surprisingly high number of candidates failed to recognise $\frac{3}{30}$ and $\frac{1}{10}$ as equivalent fractions and thus concluded that the events were not independent.

3. This proved a straightforward start to the paper. Most gave a correct tree diagram although a few oversimplified using red and not red as their outcomes and this was of no help to them in part (b). It was encouraging to see the vast majority of candidates using fractions for the probabilities and only a handful using “with replacement”.

Part (b) was not answered so well with many failing to consider both cases: blue then green and green followed by blue.

4. There were many good answers to this question. The Venn diagram was often totally correct although a number failed to subtract for the intersections and obtained value of 35, 40 and 28 instead of 31, 36 and 24 for the numbers taking two options. Parts (b) and (c) were answered very well with only a minority of candidates failing to give probabilities. Part (d) proved straightforward for those who knew what was required but some attempted complicated calculations, often involving a product of probabilities, whilst others simply gave their answer as 4/180.

5. Part (a) and part (b) were generally very well done with few candidates not knowing the correct structure of the tree diagram. A number did not fully label the tree diagram thus potentially losing the mark for the probabilities. Some candidates do not help themselves or the examiner by drawing very small diagrams. In part (b) it was pleasing to see very few candidates resorting to decimals and those who did seem to have got the message that exact equivalents are required using recurring decimals where appropriate. In part (c) many candidates demonstrated a lack of understanding of conditional probability. They could not transfer the context of the question into a formula and many still use $P(A/B)$ with no explanation as to what A and B represent. Of those who did manage to write $P(F'/L)$ many failed to see the significance of part (b)(ii).

6. Generally this question was not well answered by a large number of candidates. The terms and properties relating to probability do not seem to be fully understood, especially by weaker candidates. Part (a) was done surprising badly, with often the rest of the question fully correct. Part (c) was often correct when all else was wrong, demonstrating that candidates can use the conditional probability formula even if they do not understand it. Too few candidates write down the formula they are trying to use, which in part (d) was helpful in ascertaining if they were trying to use the correct method.

A and *B* represent. Of those who did manage to write P(*F'*/*L*) many failed to see the significance of part (b)(ii).

7. This question was not answered well. It was encouraging to see many attempting to use a diagram to help them but there were often some false assumptions made and only the better candidates sailed through this question to score full marks.

The first problem was the interpretation of the probabilities given in the question. Many thought $\frac{9}{25} = \mathrm{P}(\mathrm{E} \cap \mathrm{B})$ rather than $\mathrm{P}(\mathrm{E}|\mathrm{B})$. All possible combinations of products of two of $\frac{2}{3}, \frac{2}{5}$ and $\frac{9}{25}$ were offered for part (a) but $\frac{9}{25} = \mathrm{P}(\mathrm{E} \cap \mathrm{B})$ was the most common incorrect answer. In part (b) a variety of strategies were employed. Probably the most successful involved the use of a Venn diagram which, once part (a) had been answered could easily be constructed. Others tried using a tree diagram but there were invariably false assumptions made about $\mathrm{P}(\mathrm{E}|\mathrm{B}')$ with many thinking it was equal to $1 - \frac{9}{25}$. A few candidates assumed independence in parts (a) or (b) and did not trouble the scorers. The usual approach in part (c) involved comparing their answer from part (a) with the product of P(*E*) and P(*B*) although a few did use P(*E* | *B*) and P(*E*). Despite the question stressing that we were looking for statistical independence here, many candidates wrote about healthy living and exercise!

The large number of candidates who confused $\mathrm{P}(\mathrm{E} \cap \mathrm{B})$ and P(*E* | *B*) suggests that this is an area where students would benefit from more practice.

8. Part (a) was answered well although a small minority of candidates insisted on dividing by n (where n was usually 4). Part (b), on the other hand, caused great confusion. Some interpreted F(1.5) as E(1.5X), others interpolated between P(X = 1) and P(X = 2) and a few thought that F(1.5) was zero since X has a discrete distribution. Although the majority of candidates gained full marks in part (c) the use of notation was often poor. Statements such as Var(X) = 2 = 2 – 1 = 1 were rife and some wrote Var(X) or $\sum X^2$ when they meant E(X^2). Many candidates can now deal with the algebra of Var(X) but there were the usual errors such as 5Var(X) or 25Var(X) or –3Var(X) and the common -3^2 Var(X) which was condoned if the correct answer followed.

Part (e) was not answered well and some candidates did not attempt it. Those who did appreciate what was required often missed one or more of the possible cases or incorrectly repeated a case such as (2, 2). There were many fully correct responses though often aided by a simple table to identify the 6 cases required.

9. This question was often answered very well. The Venn diagram was usually correct although a few forgot the box and some missed the "1" outside the circles. A small minority of candidates failed to subtract the "90" from the overlaps of each pair and this meant that any attempts to follow through in later parts of the question were hopeless as their probabilities were greater than 1. Part (b) was answered well although some wrote 0.1 instead of 0.01.

Parts (c), (d) and (e) were answered well too but some candidates simply gave integer answers rather than probabilities and a few tried to multiply probabilities together. The conditional probability in part (f) was often identified but some thought that $P(C|A) = \dfrac{P(C \cap A)}{P(C)}$ and $P(C \cap A) = 0.03$ was another common error.

10. Many candidates were able to determine the correct answer for part (a) but a very common error was to multiply the two probabilities, incorrectly assuming independence. Many candidates used the Venn diagram to attain the correct solution to part (b). The most common errors were to omit a box or add a third circle. In part (c), as is often the case in this type of question, many failed to realise this was a conditional probability

11. The demand to draw a tree diagram in part (a) was probably a help to some candidates who may not otherwise have been able to get started. Part (a) was usually answered very well but a few did not interpret the conditional probabilities correctly and $P(D|A)$ was sometimes given as $\frac{3}{35}$ instead of 0.03. Sometimes $P(D \cap A)$ was confused with $P(D|A)$. Part (b) was answered well, especially part (i), although sometimes in part (ii), we saw the sum of the conditional probabilities instead of the intersections. Part (c) proved to be more of a discriminator. The correct formula was rarely quoted and even when it was seen the substitutions were often incorrect.

Throughout this question the use of correct notation was often poor: $P(C|D)$ was readily confused with $P(D|C)$ and $P(B \cap D)$ was often replaced with $P(B|D)$. It was also surprising to see how many candidates worked with percentages throughout; sometimes this led to a loss of marks if values marked on the tree diagram were not probabilities.

12. There were many fully correct solutions to this question and the ideas and techniques were clearly understood well. A few candidates misinterpreted the inequalities in part (b) and some worked throughout in decimals rather than fractions and this led to errors usually in parts (c) and (d). Some candidates did not actually carry out their calculations in part (d), they simply assumed that $21.97 - (4.47)^2$ would give them 1.97 and failed to appreciate that at least 4sf were required to obtain the printed answer. Part (e) was where most errors occurred though. Those who knew the correct formula usually obtained the correct answer, but there were a number who tried 2^2 Var(X) and some who did not know how to deal with the minus sign.

13. It was common in the Venn diagram for the value of 41 to be omitted or replaced with a zero. It seems that candidates were assuming that the hundred people in the question all possessed at least one of the attributes, i.e. they didn’t bother to add up the other values in the diagram to see that they did not come to 100. Part (b), part (c) and part (d) were generally well answered and usually followed from the values in the diagram. The conditional probability was better answered than has been the case in the past but this is still a good discriminator.

14. Parts (a) and (b) were generally well done though unexpectedly a few candidates failed to provide any sort of diagram. In part (c) relatively few candidates understood conditional probability in a fairly simple question. Poor attempts to use Bayes’ theorem usually resulted in no marks being awarded. It was a pity that only a handful of the best candidates just wrote down the answer from the tree.

15. A number of candidates seemingly had not covered Venn diagrams as many had poor diagrams or none at all. Those who knew what to do usually worked straight through and gained full marks. The mark scheme allowed a fairly generous follow through of marks which allowed some marks even after a dubious Venn diagram. Conditional probability worked better here though there were still difficulties and again more poor attempts at Bayes' theorem. In part (d) there were many good answers but a common form of error was to mistake mutually exclusive for independence. Only a few realised that they had just found P(A/B') and it wasn't P(A). Poor accuracy again caused some to lose a mark in part (c) when correct answers were truncated to 2 dps.

16. Many candidates could do this question in their heads and scored full marks. For other candidates this question caused a number of problems.

(a) The majority of candidates began their response with either a Venn diagram or a probability tree; the simpler 2-way table was only seen occasionally. Nethertheless, many candidates were able to answer part (a) correctly.

(b) Candidates who recognized the reduced sample space of 50 students were able to produce very concise solutions although a significant proportion calculated the probability of the student wearing glasses given the student is studying Arts subjects rather than the probability of the student <u>not</u> wearing glasses given the student is studying Arts subjects. Others tried to multiply probabilities without any real consideration as to whether or not the events were independent.

(c) A large number of candidates identified the need to look at all three subjects in turn and then sum. There were however, some candidates that failed to take account of the different number of students studying each subject and just added the percentages and divided by 3. A minority of candidates believed that 0.8 x 0.75 x 0.7 was sufficient.

(d) Recognition of the need for conditional probability in part (d) was good. Some candidates however wanted to divide by probability of the student studying Science subjects rather than the probability of the student being right handed. Some did not see the connection between parts (c) and (d) and tried to calculate the probability of a student being right handed again and achieved a different result to (c). The numerator in the calculation was not always correctly calculated – some candidates did not look at the 80% from science and just used 30 out of 148 as the numerator.

17. A well answered question. A fairly small minority misread the question and calculated the probability of a faulty item. The majority of candidates can draw and use tree diagrams well although a significant minority fail to label them correctly. Also, too many candidates made the mistake of putting incorrect probabilities on the second section of the tree; some were products of probabilities while some were strange fractions such as 3/85, 82/85, etc. Overall, however, many candidates gained full marks on this question

18. Candidates find it hard to translate the written information into a correct Venn Diagram, frequently forgetting to subtract one category from another. Many candidates only had 6 in the right place and 890 instead of 918 was a common error even for more able candidates. As follow through marks were allowed, they didn’t lose as many marks as they might have for these initial errors. Conditional probability is not well understood, nor was the need for use of ‘without replacement’ in part (e). Some weaker candidates still leave answers greater than 1 for probability.

19. Almost all candidates were able to produce an attempt at the Venn diagram, although the rectangle was absent in a minority of scripts. Many excellent answers were seen, but weaker candidates are unable to distinguish between P(AnB’) and P(A); the value of P(A’nB’) was also frequently omitted.

Part (b) was usually well answered, with most candidates using the algebraic form of the addition law rather than merely adding the three probabilities from their Venn diagram.

Part (c) produced a mixed response. Some very good solutions were given, but many candidates assumed that A and B’ were independent.

20. Many candidates did not realise that the phrase ‘at least 5’ meant that they had to consider three pairs of numbers – (2,3), (3,3) and (3,2). The last one was usually forgotten. A sample space diagram was all that was needed to answer this question but far too many candidates tried to use other methods and obtained the wrong answer.

21. Most candidates made a reasonable attempt to draw a Venn diagram although they did not always put the letters A, B and C in the correct order. As in previous years candidates did not recognise the meaning of ‘mutually exclusive’ and ‘independent’ and consequently were unable to answer parts (b) and (c) with any confidence. The final part of this question required candidates to know the probability rule associated with $P(A \cup C)$, use independence and then solve an equation and too many of them were unable to do so.

22. The concepts involved in this question were generally not understood by many of the candidates, particularly P(*A*Ç*B*) = P(*A*) – P(*A*Ç*B*’). But for follow through, many candidates would not have gained any of the first 7 marks. The usual confusion between ‘mutually exclusive’ and ‘independence’ was still in evidence. Candidates need to be advised that to answer this type of question they need to have the rules of probability at their fingertips.

23. Candidates found this question very accessible. Even the weaker candidates often scored highly on this question. The tree diagram was usually well drawn and part (a) was invariably correct even if the rest of the solution was wrong. In part (d) a few candidates produced an alternative answer to the one expected by finding (1 – P(at least two keys)) and this alternative was included in the mark scheme.

24. The first two marks were scored by many of the candidates, but in too many cases very few of the remaining marks were gained. Many candidates could not establish the values of X as 0, 10, 20 and 30 and they were unable to calculate corresponding probabilities. The methods for finding the mean and the standard deviation were usually known and they were often correctly applied to the distribution produced by the candidates. Too many candidates forgot to take the square root to find the standard deviation. Having struggled with part (b) candidates then could not interpret the demand in part (d).

25. Explanations in parts (a) and (b) were very poor with many candidates having no idea how to define a sample space or an event. Apart from a few slips the remaining parts of this question were well answered and many candidates gained full marks for them.

26. Many candidates could not read the given table sufficiently accurately to gain the first six marks on this question. The definition of independence was not known by many candidates and many of those that knew the definition were unable to apply it.

27. No Report available for this question.